Admission/Placement Examination 2011/2012 – MATHEMATICS (Paper 2)

Part I：Multiple-choice

1. Suppose that two nonzero real numbers satisfying $a > b$, which one of the following is correct

   (A) $a^2 > b^2$ (B) $\frac{1}{a} < \frac{1}{b}$ (C) $\log_2(a-b) > 0$ (D) $2^a > 2^b$

2. Let the set $M = \{x \mid -1 \le x < 2\}$，the set $N = \{x \mid x - k \le 0\}$. To make $M \cap N = M$ is valid, k must be taken in the set

   (A) $(-1, 2)$ (B) $[2, +\infty)$ (C) $(2, +\infty)$ (D) $[-1, 2]$

3. Let a complex number $z = 1 + i$, then $\frac{3}{z} + z^2 =$

   (A) $\frac{3+i}{2}$ (B) $\frac{3-i}{2}$ (C) $3+i$ (D) $3-i$

4. Find correct ones among the following propositions are

   ① "$a^2 + b^2 \ne 0$"equivalent to" neither a or b is 0"；

   ② "$a, b \in R^+$"is necessary and sufficient condition of "$a + b \ge 2\sqrt{ab}$".

   ③ The proposition"exists a real number x, such that $|x+2| \le 1$ and $x^2 > 16$" is false.

   ④ If the inverse-negative proposition of a proposition is true, then the proposition is true itself.

   (A) ①②③ (B) ①③ (C) ②③ (D) ①③④

5. Let $f(x) = |x-1| - |x+2|$, then $f[f(\frac{1}{2})] =$

   (A) $-\frac{1}{2}$ (B) 2 (C) 3 (D) $\frac{1}{2}$

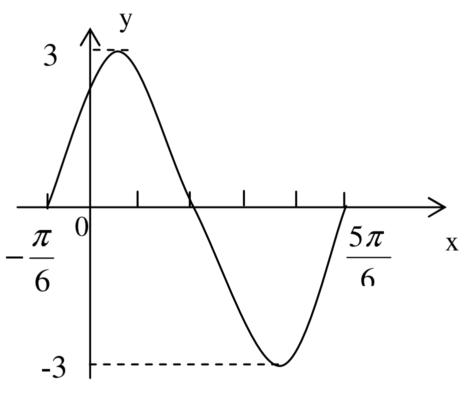
6. The figure shows the graph of a function $y = A\sin(\omega x + \varphi)$. The function should be

   (A) $y = 3\sin(x + \frac{\pi}{6})$

   (B) $y = 3\sin(x + \frac{\pi}{3})$

   (C) $y = 3\sin(2x + \frac{\pi}{6})$

   (D) $y = 3\sin(2x + \frac{\pi}{3})$



7. Known that $f(x) = ax^3 + bx + 1$, and $f(-2) = 5$, then $f(2) =$

   (A) -3 (B) 3 (C) 5 (D) -5

8. The inverse function of $y = 3^{x^2-1}\ (-1 \le x < 0)$ is

   (A) $y = -\sqrt{1 + \log_3 x}\ \left(x \ge \frac{1}{3}\right)$ (B) $y = \sqrt{1 + \log_3 x}\ \left(x \ge \frac{1}{3}\right)$

(C) $y=-\sqrt{1+\log_3 x}\left(\frac{1}{3}<x\le 1\right)$ (D) $y=\sqrt{1+\log_3 x}\left(\frac{1}{3}<x\le 1\right)$

9. $\left(\frac{1}{\sqrt{2}+1}+\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{4}+\sqrt{3}}+\cdots+\frac{1}{\sqrt{2011}+\sqrt{2010}}\right)\cdot(\sqrt{2011}+1)=$

(A) 2009 (B) 2010 (C) 2011 (D) 2012

10. The last two digits of a phone number have been forgotten, but remember the last digit is odd. The probability of success of one dial is

(A) $\frac{1}{90}$ (B) $\frac{1}{50}$ (C) $\frac{1}{45}$ (D) $\frac{1}{20}$

Part II: Calculation

1・Given $\frac{2\sin\theta+\cos\theta}{\sin\theta-3\cos\theta}=-5$, find the value of $3\cos 2\theta+4\sin 2\theta$.

2・ A circle C goes though the point $A\ (6, 5)$, and the point $B(0, 1)$, The center of the circle is on the line L_1 ： $3x+10y+9=0$. Find the equation of the circle C.

3. The sum S_n of its first n items a sequence $\{a_n\}$ is $S_n=\frac{1}{5}(a_n+3)\ (n\in N^*)$.

(1)What is a_n?

(2) Evaluate the sum $a_1+a_3+\cdots+a_{2n-1}$ of the first n odd items of the sequence`.

4・Prove $n^3+(n+1)^3+(n+2)^3$ can be divided by 9, where n is an integer.

5. A market imports a kind of good at the price 16 (MOP). Suppose that its quantity sold y (piece) is a linear function of sale price x (MOP/piece), as shown in the graph below.

(1) Write the representation of y (piece) as the function of x (MOP/piece).

(2) How much the sale price x should be to get its maximum of profit $L(x)$?

（profit = total income by sale – total expense for import）