# 澳 門 科 技 大 學
MACAU UNIVERSITY OF SCIENCE AND TECHNOLOGY

二零一七／二零一八學年入學考試
ADMISSION EXAMINATION 2017/2018

M103 數學正卷
MATHEMATICS STANDARD PAPER

<u>第一部分</u>　選擇題。

1. 若集合 $P = \{a^2, 2a+1, -5\}$，$Q = \{a+5, 1+a, 9\}$，若 $P \cap Q = \{9\}$，則 $a = ($　$)$
   A. -3　　　　B. 4　　　　C. 3　　　　D. ± 3　　　　E. -4

2. 若 $a, b, c \in R$，且 $a > b > c$，若 $a+b+c = 0$，則下列不等式恒成立的是($\quad$)
   A. $ab > bc$　　B. $ac > bc$　　　C. $a|b| > |b|c$　　D. $ab > ac$　　E. $ab \geq bc$

3. 某公司出售兩輛車，每輛車的售價均為 MOP30,000，其中一輛虧損20%，另一輛獲利20%。問該公司($\quad$)
   A. 獲利 2,500　　　　　　B. 虧損 2,500　　　　　　C. 虧損 3,000
   D. 獲利 3,000　　　　　　E. 既無獲利，也無虧損

4. x 與 $\log_2 y$ 之間的線性關係如圖所示。若 $y = ab^x$，則 $a - b = $　　$(\quad)$
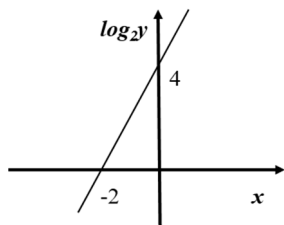
   A. 64　　　　B. 4　　　　C. -12　　　　D. 12　　　　E. 20

5. 如果 $-4x^2 + 4x + 3 > 0$，那麼 $\sqrt{4x^2 - 12x + 9} + |2x+1|$ 的值為($\quad$)
   A. $4x-2$　　B. 4　　　　C. $2-4x$　　　D. -4　　　E. -2

6. $\left(\dfrac{\lg 2+\lg 3}{\lg 3-\lg\frac{1}{2}}\right)^{7}-16^{5}\times 4^{-11}=($ $)$

    A. $\dfrac{5}{4}$ B. 0 C. $-\dfrac{3}{4}$ D. $-\dfrac{4}{3}$ E. $\dfrac{3}{4}$

7. 如果 $x^3+3x-a-3$ 能被 $x-1$ 整除，那麼 $a=($ $)$
    A. -1 B. 1 C. -2 D. 2 E. 以上皆非

8. 三個代數式的最大公因式(H.C.F.)及最小公倍式(L.C.M.)分別為 xy^2 及 $4x^4y^5z^6$。若第一個數式及第二個數式分別為 $2x^2y^4z$ 及 $4x^4y^2z^6$，則第三個數式為(   )
    A. $2xy^2z$ B. xy^2 C. $2xy^5z$ D. xy^5 E. $2xy^2z^3$

9. 已知 $\tan\alpha=-2$，則 $\dfrac{2\sin\alpha-\cos\alpha}{2\sin\alpha+\cos\alpha}=($ $)$

    A. 3 B. -3 C. $\dfrac{5}{3}$ D. $-\dfrac{5}{3}$ E. $\dfrac{3}{5}$

10. 方程 $\dfrac{2}{x^2-4}-\dfrac{1}{x(x-2)}+\dfrac{x-4}{x(x+2)}=0$ 有多少個實數解？(   )

    A. 1 B. 2 C. 3 D. 4 E. 0

11. 已知數列 a_1,a_2,a_3,a_4 成等比數列，如果 a_2,a_3 是方程 $-3x^2+5x+2=0$ 的兩個根，那麼 $a_1a_4=$
(   )
    A. $\dfrac{2}{3}$ B. $-\dfrac{3}{2}$ C. $\dfrac{5}{3}$ D. $-\dfrac{5}{3}$ E. $-\dfrac{2}{3}$

12. 一次函數 $y=-\dfrac{m}{n}x+\dfrac{1}{n}$ 的圖像同時經過第一、三、四象限的必要但不充分條件是 (   )
    A. $m>1,n<-1$ B. $mn<0$ C. $m>0,n<0$
    D. $m<0,n<0$ E. 以上都不對

13. 已知 $2\sin^2 x+\sin 2x=A\sin(\omega x+\varphi)+B$，其中 $A>0$，則 $A=($ $)$
    A. 2 B. 1 C. $\sqrt{2}$ D. $\dfrac{\sqrt{2}}{2}$ E. 3

14. 雙曲線 $\dfrac{x^2}{64}-\dfrac{y^2}{36}=1$ 上一點 P 到其右焦點的距離是 8，則點 P 到左準線的距離是
    A. $\dfrac{32}{5}$ B. $\dfrac{64}{5}$ C. 10 D. $\dfrac{96}{5}$ E. 以上都不對

15. 在 $(2x+\dfrac{1}{x^2})^6$ 的展开式中，常數項是多少？ （　）

    A. 240   B. -240 C. 0   D. -160 E. 160

Part 1   Multiple choice questions.

1. Suppose that the sets $P=\left\{a^2,2a+1,-5\right\}$ and $Q=\left\{a+5,1+a,9\right\}$, if $P\cap Q=\{9\}$, then $a=$ (　)
    A. -3 B. 4   C. 3   D. ± 3 E. -4

2. Suppose that $a,b,c\in R$, $a>b>c$ and $a+b+c=0$, which one of the following is correct (　)
    A. $ab>bc$ B. $ac>bc$ C. $a|b|>|b|c$ D. $ab>ac$ E. $ab\geq bc$

3. A company sold two cars and the price of each car is MOP30,000. One loss 20%, another profit 20%. Which one of the following statements is true? (　)
    A. gain 2,500   B. lose 2,500   C. lose 3,000
    D. gain 3,000   E. Neither gain nor lose

4. The linear relationship between x and $\log_2 y$ is shown in fig. If $y=ab^x$, then $a-b=$ (　)
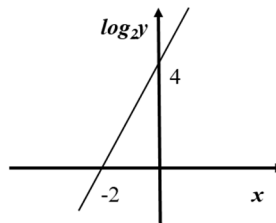
    A. 64   B. 4   C. -12 D. 12   E. 20

5. If $-4x^2+4x+3>0$, then $\sqrt{4x^2-12x+9}+|2x+1|=$ (　)
    A. $4x-2$ B. 4   C. $2-4x$ D. -4 E. -2

6. $\left(\dfrac{\lg 2+\lg 3}{\lg 3-\lg\dfrac{1}{2}}\right)^7-16^5\times 4^{-11}=$ (　)
    A. $\dfrac{5}{4}$ B. 0   C. $-\dfrac{3}{4}$ D. $-\dfrac{4}{3}$ E. $\dfrac{3}{4}$

7. If $x^3+3x-a-3$ is divisible by $x-1$, then $a=$ (　)
    A. -1 B. 1   C. -2 D. 2   E. none of the above

8. Assume that xy^2 and $4x^4y^5z^6$ are the Highest Common Factor (H.C.F.) and the Least Common Multiple (L.C.M.) of three algebraic expressions respectively. If two of them are $2x^2y^4z$ and $4x^4y^2z^6$, then the third algebraic expression is (   )

   A. $2xy^2z$ B. xy^2 C. $2xy^5z$ D. xy^5 E. $2xy^2z^3$

9. Given $\tan\alpha = -2$, then $\dfrac{2\sin\alpha - \cos\alpha}{2\sin\alpha + \cos\alpha} = ($ $)$

   A. 3   B. -3 C. $\dfrac{5}{3}$ D. $-\dfrac{5}{3}$ E. $\dfrac{3}{5}$

10. How many real root(s) of the equation $\dfrac{2}{x^2-4} - \dfrac{1}{x(x-2)} + \dfrac{x-4}{x(x+2)} = 0$?(   )

    A. 1   B. 2   C. 3   D. 4   E. 0

11. Suppose that a_1, a_2, a_3, a_4 is a geometric progression, a_2 and a_3 are roots of the equation $-3x^2 + 5x + 2 = 0$, then $a_1a_4 = ($ $)$

    A. $\dfrac{2}{3}$ B. $-\dfrac{3}{2}$ C. $\dfrac{5}{3}$ D. $-\dfrac{5}{3}$ E. $-\dfrac{2}{3}$

12. (   ) is a necessary not sufficient condition of the proposition, that is, the graph of the linear function $y = -\dfrac{m}{n}x + \dfrac{1}{n}$ passes through the first, third and fourth quadrants.

    A. $m > 1, n < -1$ B. $mn < 0$ C. $m > 0, n < 0$
    D. $m < 0, n < 0$ E. none

13. Suppose that $2\sin^2 x + \sin 2x = A\sin(\omega x + \varphi) + B$, where $A > 0$, then $A = ($ $)$

    A. 2   B. 1   C. $\sqrt{2}$ D. $\dfrac{\sqrt{2}}{2}$ E. 3

14. Given hyperbola $\dfrac{x^2}{64} - \dfrac{y^2}{36} = 1$, P is a point on the hyperbola, the distance from P to its right focus is 8, then the distance from P to its left directrix is

    A. $\dfrac{32}{5}$ B. $\dfrac{64}{5}$ C. 10   D. $\dfrac{96}{5}$ E. none

15. In the expansion of $(2x + \dfrac{1}{x^2})^6$, what is the constant term ? (   )

    A. 240   B. -240 C. 0   D. -160 E. 160

第二部分 解答題。**Part II** **Problem-solving questions.**

1. 把 $\dfrac{-x^3+x+1}{x^2+x-2}$ 化為部分分式。 (8 分)

   Find the partial fraction decomposition of $\dfrac{-x^3+x+1}{x^2+x-2}$. (8 marks)

2. 已知方程 $\dfrac{x^2}{5-k}+\dfrac{y^2}{k-3}=1$ 表示橢圓，求 k 的取值範圍。 (6 分)

   Assume that $\dfrac{x^2}{5-k}+\dfrac{y^2}{k-3}=1$ is an ellipse. Find the range of values of k . (6 marks)

3. 從 5 名男生和 3 名女生中任選 3 人參加歌唱比賽，求以下各項的概率。
   (a) 3 人都是男生。 (4 分)
   (b) 3 人中至少有 1 名男生。 (4 分)

   Suppose three persons are randomly chosen from five boys and three girls to compete
   in a singing contest, what's the probability for each of the following events?
   (a) all the three boys are chosen. (4 marks)
   (b) at least one boy is chosen. (4 marks)

4. 等差數列 $\{a_n\}$ 的前 n 項和 S_n 滿足 $S_5=30, S_{10}=110$。數列 $\{b_n\}$ 的前 n 項和 M_n 滿足
   $b_1=1, b_{n+1}-2M_n=1$

   (a) 求 S_n 的表達式。 (4 分)
   (b) 求 M_n 的表達式。 (6 分)

   Let $\{a_n\}_{n\geq 1}$ be an arithmetic progression, S_n denotes the sum of first n terms of
   $\{a_n\}_{n\geq 1}$ satisfy with $S_5=30, S_{10}=110$. Let M_n denote the sum of first n terms of
   $\{b_n\}_{n\geq 1}$ and satisfy with $b_1=1, b_{n+1}-2M_n=1$.

   (a) Find S_n . (4 marks)
   (b) Find M_n . (6 marks)

5. 用數學歸納法證明 $1+2+3+\cdots+n^2=\dfrac{n^2(n^2+1)}{2}$, $(n\in N^*)$ 。 (8 分)

   Prove $1+2+3+\cdots+n^2=\dfrac{n^2(n^2+1)}{2}$ $(n\in N^*)$ by mathematic induction. (8 marks)