



澳門科技大學  
MACAU UNIVERSITY OF SCIENCE AND TECHNOLOGY

二零一九年入學考試  
JOINT ADMISSION EXAMINATION 2019

M103 數學正卷  
MATHEMATICS STANDARD PAPER

第一部分 選擇題。請選出每題之最佳答案。

1. 設集合  $A = \{x: x^2 - 4x + 3 > 0\}$ ,  $B = \{x: 2x - 3 < 0\}$ , 則  $A \cap B =$   
A.  $(-\infty, -\frac{3}{2})$       B.  $(1, \frac{3}{2})$       C.  $(1, 3)$       D.  $(\frac{3}{2}, 3)$       E.  $(-\infty, 1)$
2. 若  $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$ ,  $x^2 + y^2 + z^2 = 4$ , 則  $x + y + z =$   
A. 2      B. -2      C. -2或2      D. 4      E. 4或-4
3. 不等式  $|2x + 1| > 1 + x$  的解為  
A.  $x < -\frac{2}{3}$  或  $x > 0$       B.  $x < -\frac{2}{3}$   
C.  $x > 0$       D.  $-\frac{2}{3} < x < 0$   
E.  $x < 0$
4. 已知雙曲綫  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  的一條漸近綫方程為  $y = \frac{4}{3}x$ , 則雙曲綫離心率為  
A.  $\frac{4}{3}$       B.  $\frac{5}{4}$       C.  $\frac{3}{2}$       D.  $\frac{5}{3}$       E. 以上皆非
5. 有3位女生、4位男生排成一排照相, 其中女生必須在一起的排法共有多少種?  
A. 6      B. 720      C. 120      D. 12      E. 以上皆非
6. 二項式  $(x^2 + \frac{1}{x^4})^3$  的展開式中的常數項為  
A. 1      B. -3      C. -1      D. 3      E. 0
7.  $(4m - n)^2 - (4m + n)^2 =$   
A. 0      B.  $2n^2$       C.  $-16mn$       D.  $16mn$       E.  $-8mn$

8. 已知某圓的半徑為10cm，如果一條直線和該圓的圓心的距離為10cm，那麼這條直線和這個圓的位置關係為

- A. 相切      B. 相離      C. 相交或相離      D. 相交      E. 以上皆非

9. 為得到函數 $y = \sin x$ 的圖像，可將函數 $y = \cos x$ 的圖像

- A. 向右平移 $\pi$ 個單位長度      B. 向左平移 $\pi$ 個單位長度  
C. 向右平移 $\frac{\pi}{2}$ 個單位長度      D. 向左平移 $\frac{\pi}{2}$ 個單位長度  
E. 以上皆非

10. 若變量 $x, y$ 滿足約束條件 $\begin{cases} x \leq 2 \\ y \leq 2 \\ x + y \geq 2 \end{cases}$ ，則 $x + 2y$ 的最大值為

- A. 2      B. 3      C. 4      D. 5      E. 6

11. 在等比數列 $\{b_n\}_{n \geq 1}$ 中 $b_2 = 2$ ，則 $b_1 b_2 b_3 =$

- A. 16      B. 8      C. 4      D. 2      E. 32

12. 已知 $\tan \alpha = -2$ ， $\tan(\alpha + \beta) = 1$ ，則 $\tan \beta =$

- A. 1      B.  $-\frac{1}{3}$       C.  $\frac{1}{3}$       D. 3      E. -3

13. 已知 $\alpha, \beta$ 是一元二次方程 $2x^2 - 3x - 5 = 0$ 的兩個實根，則 $|\alpha - \beta| =$

- A. 0      B.  $\frac{3}{2}$       C.  $\frac{5}{2}$       D.  $\frac{7}{2}$       E. 以上皆非

14.  $\sqrt{17 - 12\sqrt{2}} =$

- A.  $3 - 2\sqrt{2}$       B.  $3 + 2\sqrt{2}$       C.  $2\sqrt{2} - 3$       D. 0      E. 以上皆非

15. 若 $\frac{2}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}$ ，則 $AB =$

- A. 0      B. -1      C. 1      D. 2      E. 以上皆非

**Part I Multiple choice questions. Choose the best answer for each question.**

1. If  $A = \{x: x^2 - 4x + 3 > 0\}$ ,  $B = \{x: 2x - 3 < 0\}$ , then  $A \cap B =$   
A.  $(-\infty, -\frac{3}{2})$       B.  $(1, \frac{3}{2})$       C.  $(1, 3)$       D.  $(\frac{3}{2}, 3)$       E.  $(-\infty, 1)$
2. If  $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$ ,  $x^2 + y^2 + z^2 = 4$ , then  $x + y + z =$   
A. 2      B. -2      C. -2 or 2      D. 4      E. 4 or -4
3. The solution of the inequality  $|2x + 1| > 1 + x$  is  
A.  $x < -\frac{2}{3}$  or  $x > 0$       B.  $x < -\frac{2}{3}$   
C.  $x > 0$       D.  $-\frac{2}{3} < x < 0$   
E.  $x < 0$
4. If one of the asymptotic equations of the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  is  $y = \frac{4}{3}x$ , then the eccentricity of the hyperbola is  
A.  $\frac{4}{3}$       B.  $\frac{5}{4}$       C.  $\frac{3}{2}$       D.  $\frac{5}{3}$       E. None of the above
5. There are 3 girls and 4 boys in a row to take pictures, and 3 girls are required to stand together. How many different ways can they stand?  
A. 6      B. 720      C. 120      D. 12      E. None of the above
6. The constant in the expansion of  $(x^2 + \frac{1}{x^4})^3$  is  
A. 1      B. -3      C. -1      D. 3      E. 0
7.  $(4m - n)^2 - (4m + n)^2 =$   
A. 0      B.  $2n^2$       C.  $-16mn$       D.  $16mn$       E.  $-8mn$
8. Given that the radius of a circle is 10cm. If the distance between a line and the center of the circle is 10cm, then the position relationship between the line and the circle is  
A. tangency      B. separation  
C. intersection or separation      D. intersection  
E. None of the above
9. To obtain the graph of the function  $y = \sin x$ , we can shift the graph of the function  $y = \cos x$   
A. to the right by  $\pi$  unit      B. to the left by  $\pi$  unit  
C. to the right by  $\frac{\pi}{2}$  unit      D. to the left by  $\frac{\pi}{2}$  unit  
E. None of the above

10. If the variables  $x, y$  satisfy the constraints  $\begin{cases} x \leq 2 \\ y \leq 2 \\ x + y \geq 2 \end{cases}$ , then the maximum of  $x + 2y$  is  
 A. 2                      B. 3                      C. 4                      D. 5                      E. 6
11. In the geometric sequence  $\{b_n\}_{n \geq 1}$ ,  $b_2 = 2$ , then  $b_1 b_2 b_3 =$   
 A. 16                      B. 8                      C. 4                      D. 2                      E. 32
12. If  $\tan \alpha = -2$ ,  $\tan(\alpha + \beta) = 1$ , then  $\tan \beta =$   
 A. 1                      B.  $-\frac{1}{3}$                       C.  $\frac{1}{3}$                       D. 3                      E.  $-3$
13. If  $\alpha, \beta$  are two real roots of the equation  $2x^2 - 3x - 5 = 0$ , then  $|\alpha - \beta| =$   
 A. 0                      B.  $\frac{3}{2}$                       C.  $\frac{5}{2}$                       D.  $\frac{7}{2}$                       E. None of the above
14.  $\sqrt{17 - 12\sqrt{2}} =$   
 A.  $3 - 2\sqrt{2}$                       B.  $3 + 2\sqrt{2}$                       C.  $2\sqrt{2} - 3$                       D. 0                      E. None of the above
15. If  $\frac{2}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}$ , then  $AB =$   
 A. 0                      B.  $-1$                       C. 1                      D. 2                      E. None of the above

**第二部分 解答題。 Part II Problem-solving questions.**

1. 已知函數  $f(x) = (\sin x + \cos x)^2 + \cos 2x$ 。

- (1) 求  $f(x)$  的最小正週期。 (4 分)  
(2) 求  $f(x)$  在閉區間  $[0, \pi]$  上的最大值。 (4 分)

Let  $f(x) = (\sin x + \cos x)^2 + \cos 2x$ .

- (1) Find the minimum positive period of  $f(x)$ . (4 marks)  
(2) Find the maximum of  $f(x)$  on the closed interval  $[0, \pi]$ . (4 marks)

2. 設  $\{a_n\}_{n \geq 1}$  是一個等差數列，且  $a_1 = \log_2 3$ ， $a_2 + a_3 = 5 \log_2 3$ 。

- (1) 求  $\{a_n\}_{n \geq 1}$  的通項公式。 (4 分)  
(2) 求  $2^{a_1} + 2^{a_2} + \dots + 2^{a_n}$ 。 (4 分)

Let  $\{a_n\}_{n \geq 1}$  be an arithmetic sequence, and  $a_1 = \log_2 3$ ,  $a_2 + a_3 = 5 \log_2 3$ .

- (1) Find the general formula of  $\{a_n\}_{n \geq 1}$ . (4 marks)  
(2) Find  $2^{a_1} + 2^{a_2} + \dots + 2^{a_n}$ . (4 marks)

3. 解方程  $4^x + 2^x - 6 = 0$ . (8 分)

Solve the equation  $4^x + 2^x - 6 = 0$ . (8 marks)

4. 已知橢圓  $x^2 + \frac{y^2}{4} = \frac{1}{4}$  及直線  $y = x + b$ 。

- (1) 當  $b$  為何值時，直線與橢圓無公共點？ (4 分)  
(2) 若直線被橢圓截得的弦長為  $\frac{2}{5}\sqrt{10}$ ，求  $b$  的值。 (4 分)

Assume that the equation of ellipse is  $x^2 + \frac{y^2}{4} = \frac{1}{4}$  and the equation of line is  $y = x + b$ .

- (1) For which value of  $b$ , the line and the ellipse have no common point. (4 marks)  
(2) If the line is truncated by the ellipse and the string length is  $\frac{2}{5}\sqrt{10}$ , find the value of  $b$ . (4 marks)

5. 盒中裝有兩張外形一樣的卡片，其中一張卡片上寫有數字"1"，另一張卡片上寫有數字"2"。現在做有放回地隨機抽取 3 次，每次抽取一張卡片。

- (1) 問一共有多少種不同的結果？ (4 分)  
(2) 求 3 次抽取卡片上數字總和為 5 的概率。 (4 分)

A box contains two cards of the same shape, one with the number "1" on it and the other one with the number "2" on it. Now 3 cards are drawn randomly from the box with replacement, one card at a time.

- (1) How many different outcomes are there? (4 marks)  
(2) Find the probability that the sum of the number on the 3 cards is 5. (4 marks)