



澳門科技大學  
MACAU UNIVERSITY OF SCIENCE AND TECHNOLOGY

二零二五年入學考試  
ADMISSION EXAMINATION 2025

M103 數學 MATHEMATICS

**第一部份 選擇題。請選出每題之最佳答案。**

1. 若  $M = \{x \mid x^2 + 5x + 6 > 0\}$ ,  $N = \{x \mid x \geq 4\}$ , 則  $M \cap N = ( \quad )$ 。
- A.  $(-2, 4]$                       B.  $(-3, 4]$                       C.  $[-3, \infty)$                       D.  $(-2, \infty)$                       E.  $[4, \infty)$
2. 已知實數  $a < b$ , 則下列不等式正確的是 (      )。
- A.  $\frac{5}{a} < \frac{5}{b}$                       B.  $a^2 < b^2$                       C.  $(\frac{1}{2})^a > (\frac{1}{2})^b$
- D.  $\ln a < \ln b$                       E.  $\sin a < \sin b$
3. 已知  $\sin x - \cos x = \frac{1}{3}$ , 則  $\sin 2x = ( \quad )$ 。
- A.  $\frac{3}{4}$                       B.  $\frac{1}{4}$                       C.  $\frac{8}{9}$                       D.  $\frac{\sqrt{3}}{2}$                       E.  $\frac{\sqrt{2}}{3}$
4. 已知函數  $f(x) = \log_3(3x + \sqrt{9x^2 + 5a})$  為奇函數, 那麼  $a = ( \quad )$ 。
- A. 0                      B. 3                      C.  $\frac{1}{5}$
- D.  $\frac{3}{5}$                       E.  $\frac{9}{5}$
5. 在  $\triangle ABC$  中,  $AB = 4$ ,  $AC = 5$ ,  $BC = \sqrt{21}$ 。則  $\triangle ABC$  的面積為 (      )。
- A.  $5\sqrt{3}$                       B.  $\frac{5\sqrt{3}}{2}$                       C. 5                      D.  $\frac{5}{2}$                       E.  $\frac{3\sqrt{3}}{8}$
6.  $(x^2 + \frac{2}{x})^5$  的展開式中  $x^4$  項的係數是 (      )。
- A. 30                      B. 35                      C. 14
- D. 40                      E. 10
7. 等差數列  $\{a_n\}_{n=1}^{\infty}$  前  $n$  項和為  $S_n$ 。若  $S_{10} = 40$ ,  $a_6 = 5$ , 則  $a_{10} = ( \quad )$ 。
- A. 10                      B. 13                      C. 15                      D. 16                      E. 30
8. 已知函數  $f(x)$  是偶函數,  $g(x)$  是奇函數, 且  $g(x) = f(x+1)$ , 則  $f(2025) = ( \quad )$ 。
- A. 0                      B. 1                      C. 2                      D. 2025                      E. -2025

9. 設樣本數據  $x_1, x_2, \dots, x_{15}$  的均值和方差分別為 1 和 4，那麼數據  $2x_1 - 1, 2x_2 - 1, \dots, 2x_{15} - 1$  的均值和方差分別為 ( )。
- A. 1 和 16                      B. 1 和 4                      C. 2 和 16                      D. 2 和 4                      E. 1 和 7
10. 函數  $f(x) = \sin(2x + \omega)$  (其中  $0 \leq \omega < \pi$ ) 的圖像向左平移  $\frac{\pi}{6}$  後關於  $y$  軸對稱。則  $\omega =$  ( )。
- A.  $\frac{\pi}{6}$                       B.  $\frac{\pi}{3}$                       C.  $\frac{\pi}{2}$                       D.  $\frac{2\pi}{3}$                       E.  $\frac{5\pi}{6}$
11.  $M$  是拋物線  $y^2 = 4x$  上的一點， $N$  是圓  $(x - 3)^2 + y^2 = 1$  上的點，則  $|MN|$  的最小值為 ( )。
- A. 3                      B.  $\sqrt{3}$                       C.  $\sqrt{2} - 1$                       D.  $2\sqrt{2} - 1$                       E. 1
12. 不等式  $x^2 + ax + 1 > 0$  (其中參數  $a \in \mathbb{R}$ ) 對於  $x \geq 1$  恆成立。則  $a$  的取值範圍是 ( )。
- A.  $(-2, 2)$                       B.  $(-2, 2) \cup (2, \infty)$                       C.  $(-\infty, 2)$                       D.  $[2, \infty)$                       E.  $(-2, \infty)$
13. 3 本不同的地理書，2 本不同的數學書。從中任意取出 2 本，取出的恰好都是數學書的概率為 ( )。
- A.  $\frac{1}{5}$                       B.  $\frac{1}{10}$                       C.  $\frac{1}{3}$                       D.  $\frac{1}{2}$                       E.  $\frac{3}{5}$
14. 雙曲線  $\frac{x^2}{4} - y^2 = 1$  的漸近線為 ( )。
- A.  $x \pm 2y = 0$                       B.  $2x \pm y = 0$                       C.  $4x \pm y = 0$                       D.  $x \pm 4y = 0$                       E.  $x \pm \sqrt{5}y = 0$
15. 函數  $f(x) = (a - 2)^x + 3$  (其中  $a > 2, a \neq 3$ ) 的圖像一定過點 ( )。
- A.  $(1, 2)$                       B.  $(0, 4)$                       C.  $(3, 4)$                       D.  $(0, 3)$                       E.  $(2, 3)$

**Part I Multiple choice questions. Choose the *best answer* for each question.**

1. Let  $M = \{x \mid x^2 + 5x + 6 > 0\}$  and  $N = \{x \mid x \geq 4\}$ . Then  $M \cap N = ( \quad )$ .  
A.  $(-2, 4]$                       B.  $(-3, 4]$                       C.  $[-3, \infty)$                       D.  $(-2, \infty)$                       E.  $[4, \infty)$
2. Given real numbers  $a < b$ , which of the following inequalities is correct?  
A.  $\frac{5}{a} < \frac{5}{b}$                                       B.  $a^2 < b^2$                                       C.  $(\frac{1}{2})^a > (\frac{1}{2})^b$   
D.  $\ln a < \ln b$                                       E.  $\sin a < \sin b$
3. Given  $\sin x - \cos x = \frac{1}{3}$ , then  $\sin 2x = ( \quad )$ .  
A.  $\frac{3}{4}$                                       B.  $\frac{1}{4}$                                       C.  $\frac{8}{9}$                                       D.  $\frac{\sqrt{3}}{2}$                                       E.  $\frac{\sqrt{2}}{3}$
4. If the function  $f(x) = \log_3(3x + \sqrt{9x^2 + 5a})$  is odd, then  $a = ( \quad )$ .  
A. 0                                      B. 3                                      C.  $\frac{1}{5}$   
D.  $\frac{3}{5}$                                       E.  $\frac{9}{5}$
5. In  $\triangle ABC$  with  $AB = 4$ ,  $AC = 5$ , and  $BC = \sqrt{21}$ , the area of the triangle is  $( \quad )$ .  
A.  $5\sqrt{3}$                                       B.  $\frac{5\sqrt{3}}{2}$                                       C. 5                                      D.  $\frac{5}{2}$                                       E.  $\frac{3\sqrt{3}}{8}$
6. The coefficient of the  $x^4$  term in the expansion of  $(x^2 + \frac{2}{x})^5$  is  $( \quad )$ .  
A. 30                                      B. 35                                      C. 14  
D. 40                                      E. 10
7. For an arithmetic sequence  $\{a_n\}_{n=1}^{\infty}$  with partial sum  $S_n$ , given  $S_{10} = 40$  and  $a_6 = 5$ , then  $a_{10} = ( \quad )$ .  
A. 10                                      B. 13                                      C. 15                                      D. 16                                      E. 30
8. Given  $f(x)$  is an even function and  $g(x)$  is an odd function with  $g(x) = f(x+1)$ , then  $f(2025) = ( \quad )$ .  
A. 0                                      B. 1                                      C. 2                                      D. 2025                                      E.  $-2025$
9. Let sample data  $x_1, x_2, \dots, x_{15}$  have mean 1 and variance 4. Then the mean and variance of  $2x_1 - 1, 2x_2 - 1, \dots, 2x_{15} - 1$  are respectively  $( \quad )$ .  
A. 1 and 16                                      B. 1 and 4                                      C. 2 and 16                                      D. 2 and 4                                      E. 1 and 7

10. The graph of  $f(x) = \sin(2x + \omega)$  ( $0 \leq \omega < \pi$ ) is shifted left by  $\frac{\pi}{6}$  and becomes symmetric about the  $y$ -axis. Then  $\omega =$  ( ).
- A.  $\frac{\pi}{6}$                       B.  $\frac{\pi}{3}$                       C.  $\frac{\pi}{2}$                       D.  $\frac{2\pi}{3}$                       E.  $\frac{5\pi}{6}$
11. Let  $M$  be a point on the parabola  $y^2 = 4x$  and  $N$  a point on the circle  $(x - 3)^2 + y^2 = 1$ . The minimum value of  $|MN|$  is ( ).
- A. 3                      B.  $\sqrt{3}$                       C.  $\sqrt{2} - 1$                       D.  $2\sqrt{2} - 1$                       E. 1
12. The inequality  $x^2 + ax + 1 > 0$  (with parameter  $a \in \mathbb{R}$ ) holds for all  $x \geq 1$ . Then the range of  $a$  is ( ).
- A.  $(-2, 2)$                       B.  $(-2, 2) \cup (2, \infty)$                       C.  $(-\infty, 2)$                       D.  $[2, \infty)$                       E.  $(-2, \infty)$
13. There are 3 different geography books and 2 different math books. The probability that two randomly selected books are both math books is ( ).
- A.  $\frac{1}{5}$                       B.  $\frac{1}{10}$                       C.  $\frac{1}{3}$                       D.  $\frac{1}{2}$                       E.  $\frac{3}{5}$
14. The asymptotes of the hyperbola  $\frac{x^2}{4} - y^2 = 1$  are ( ).
- A.  $x \pm 2y = 0$                       B.  $2x \pm y = 0$                       C.  $4x \pm y = 0$                       D.  $x \pm 4y = 0$                       E.  $x \pm \sqrt{5}y = 0$
15. The graph of  $f(x) = (a - 2)^x + 3$  ( $a > 2, a \neq 3$ ) must pass through the point ( ).
- A.  $(1, 2)$                       B.  $(0, 4)$                       C.  $(3, 4)$                       D.  $(0, 3)$                       E.  $(2, 3)$

**第二部份 解答题。 Part II Problem-solving questions.**

1. 已知  $f(x) = 2\sqrt{3} \sin x \cos x + 2 \cos^2 x - 1$ 。

(a) 求  $f(\frac{\pi}{6})$  的值。 (4 分)

(b) 求  $f(x)$  遞增區間。 (4 分)

Let  $f(x) = 2\sqrt{3} \sin x \cos x + 2 \cos^2 x - 1$ .

(a) Find the value of  $f(\frac{\pi}{6})$ . (4 marks)

(b) Find the increasing intervals of  $f(x)$ . (4 marks)

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2. 已知正項數列  $\{a_n\}_{n=1}^{\infty}$  的前  $n$  項和為  $S_n$ ，且滿足  $6S_n = a_n^2 + 3a_n$ 。

(a) 求  $a_1$  和  $a_2$ 。 (4 分)

(b) 求數列  $\{a_n\}_{n=1}^{\infty}$  的通項公式。 (4 分)

Let  $S_n$  be the sum of the first  $n$  terms of the positive sequence  $\{a_n\}_{n=1}^{\infty}$ , and it satisfies  $6S_n = a_n^2 + 3a_n$ .

(a) Find  $a_1$  and  $a_2$ . (4 marks)

(b) Find the general formula for the sequence  $\{a_n\}_{n=1}^{\infty}$ . (4 marks)

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3. 設  $A = \{x \mid x^2 - px - 2 = 0\}$  ,  $B = \{x \mid x^2 + qx + r = 0\}$  ,  $A \cap B = \{-2\}$  ,  $A \cup B = \{-2, -1, 1\}$  。
- (a) 求  $p$  的值。 (3 分)
- (b) 求  $q$  的值。 (3 分)
- (c) 求  $r$  的值。 (2 分)

Let  $A = \{x \mid x^2 - px - 2 = 0\}$ ,  $B = \{x \mid x^2 + qx + r = 0\}$ ,  $A \cap B = \{-2\}$ ,  $A \cup B = \{-2, -1, 1\}$ .

- (a) Find the value of  $p$ . (3 marks)
- (b) Find the value of  $q$ . (3 marks)
- (c) Find the value of  $r$ . (2 marks)

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4. 已知  $\triangle ABC$  的內角  $A, B, C$  的對邊分別為  $a, b, c$  ,  $\frac{2a}{\cos A} = \frac{3c - 2b}{\cos B}$  ,  $b = \sqrt{5} \sin B$  , 求  $a$  。 (8 分)

Given that the angles  $A, B, C$  of  $\triangle ABC$  have opposite sides  $a, b, c$  respectively, and satisfy  $\frac{2a}{\cos A} = \frac{3c - 2b}{\cos B}$  ,  $b = \sqrt{5} \sin B$ . Find  $a$ . (8 marks)

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5. 已知正項級數  $\{a_n\}_{n=1}^{\infty}$  中， $a_1 = 1$ ， $a_{n+1} = 1 + \frac{a_n}{1 + a_n}$  ( $n \in \mathbb{N}^+$ )。用數學歸納法證明： $a_n < a_{n+1}$ ， $n \in \mathbb{N}^+$ 。  
(8 分)

In the positive sequence  $\{a_n\}_{n=1}^{\infty}$ ,  $a_1 = 1$ ,  $a_{n+1} = 1 + \frac{a_n}{1 + a_n}$  ( $n \in \mathbb{N}^+$ ). Using mathematical induction to prove  $a_n < a_{n+1}$ ,  $n \in \mathbb{N}^+$ .  
(8 marks)

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