



澳門四高校聯合入學考試(語言科及數學科)

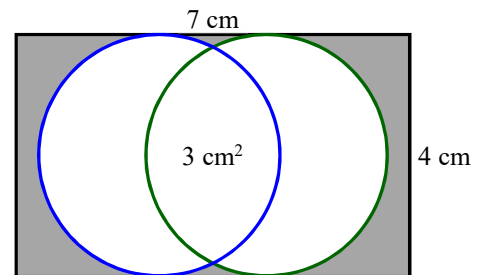
Joint Admission Examination for Macao Four Higher Education Institutions (Languages and Mathematics)

2021 試題及參考答案 2021 Examination Paper and Suggested Answer

數學正卷 Mathematics Standard Paper

第一部份 選擇題。請選出每題之最佳答案。

1. 若集合 $P = \{1, 2, 3, 5, 7, 11\}$, $Q = \{x: x^2 - 15x + 36 < 0\}$, 則 $P \cap Q$ 中元素的個數為
A. 2 B. 3 C. 4 D. 5 E. 1
2. 已知瑪麗和約翰兩人分別要用 4 小時及 3 小時完成某件工作。若他們一起做, 需要多少小時才能完成 5 件相同的工作?
A. $32/5$ B. $35/4$ C. $35/2$ D. $20/3$ E. $60/7$
3. 有一等差數列前 n 項之總和為 n^2 , 求此數列的第 10 項。
A. 19 B. 21 C. 28 D. 31 E. 40
4. 若對所有實數 x , $y = mx^2 + 6x + 3m$ 都為正數, 求 m 的取值範圍。
A. $0 < m < \sqrt{3}$ B. $m > \sqrt{3}$ C. $-\sqrt{3} < m < \sqrt{3}$
D. $-\sqrt{3} < m < 0$ E. 以上皆非
5. 已知 $-2x^2 + 3x - 7 = 0$ 的根為 α 和 β 。下列哪一個方程的根為 $\frac{1}{\alpha}$ 和 $\frac{1}{\beta}$?
A. $x^2 - 3x + 7 = 0$ B. $7x^2 - 3x + 2 = 0$ C. $7x^2 + 3x + 2 = 0$
D. $2x^2 - 3x - 7 = 0$ E. 以上皆非
6. 右圖中兩個半徑相等的圓與長方形的上、下邊相切。若長方形的長和寬分別為 7 cm 及 4 cm, 而兩個圓的相交部份有面積 3 cm^2 , 陰影部份面積為多少 cm^2 ?
A. $31 - 8\pi$ B. $27 - 8\pi$
C. $27 - 4\pi$ D. $21 - 4\pi$
E. 以上皆非
7. 若方程式 $9^{-x^2} - 4 \cdot 3^{-x^2} = k$ 有實數解, 下列哪個一定成立?
A. $k > 0$ B. $-4 \leq k \leq 1$ C. $-3 \leq k < 0$ D. $0 < k \leq 3$ E. 以上皆非
8. 設 $f(x) = -16x^3 - mx - m$ 。若 $f(x)$ 能被 $2x + 1$ 整除, 求 m 之值。
A. -1 B. 1 C. 2 D. 4 E. 6
9. 103^{10} 的十位數字 (右面起計第二個數字; 例如 43128 的十位數字是 2) 是
A. 2 B. 3 C. 4 D. 7 E. 以上皆非

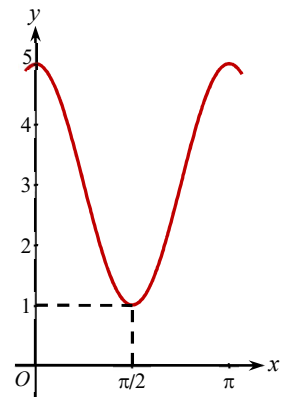


10. 若 $\log_4 x = y - 3$ 及 $2(\log_4 x)^2 = 4 - y$ ，則 $x =$

- A. $\frac{1}{4}$ 或 2 B. $\frac{1}{2}$ 或 4 C. $\frac{7}{2}$ 或 2 D. $\frac{1}{4}$ 或 $\frac{7}{2}$ E. 2 或 4

11. 右圖中所示為 _____ 的圖像。

- A. $y = 3 + 2 \cos \frac{x}{2}$
B. $y = 3 + 2 \cos 2x$
C. $y = 3 + 2 \cos x$
D. $y = 1 + 2 \cos \frac{x}{2}$
E. $y = 1 + 2 \cos 2x$



12. 已知點 $P(-1, -3)$ 和 $Q(5, -1)$ ，則 PQ 的垂直平分線的方程為

- A. $x + 3y - 4 = 0$ B. $x - 3y + 4 = 0$ C. $x + 3y + 4 = 0$
D. $3x - y - 4 = 0$ E. $3x + y - 4 = 0$

13. 若一組數據 $x_1, x_2, \dots, x_n$ 的平均數和方差分別為 1 和 0.01，則數據 $10x_1, 10x_2, \dots, 10x_n$ 的平均數和方差分別是

- A. 1 和 0.01 B. 10 和 0.1 C. 1 和 1 D. 10 和 1 E. 100 和 1

14. $\frac{\sqrt{140} - \sqrt{132}}{\sqrt{35} + \sqrt{33}} =$

- A. $68 - 2\sqrt{1155}$ B. $68 - \sqrt{1155}$ C. $(34 - \sqrt{1155})/2$
D. $34 - \sqrt{1155}$ E. $68 + 2\sqrt{1155}$

15. 若 $\frac{5}{a} + \frac{4}{b} = 3$ ($a, b > 0$)，則 ab 的最小值為

- A. $\frac{20}{9}$ B. $\frac{20}{3}$ C. $\frac{80}{9}$ D. $\frac{80}{3}$ E. $\frac{\sqrt{20}}{3}$

第二部份 解答題。

1. 書架上有中文書 4 本、英文書 2 本、數學書 3 本。

(a) 從這書架上隨機地選取 3 本書。求取得中、英、數各一本的概率。(3 分)

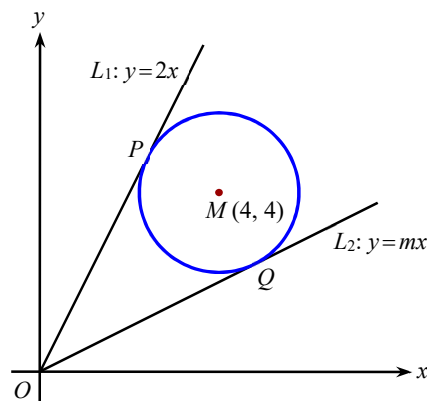
(b) 將這九本書隨機地重新排列。求同類書籍排在一起的概率。(5 分)

[注：以最簡分數表示 (a) 和 (b) 的答案。]

2. 右圖中，一個以 $M(4, 4)$ 為圓心的圓與 $L_1: y=2x$ 和 $L_2: y=mx$ 兩條直線相切。兩條直線對圓的切點分別為 P 和 Q 。

(a) 求圓的方程。(4 分)

(b) 求 m 的值。(4 分)



3. 若 $x, y > 0$ 及 $y^2 - 2myx + x^2 = a^2$ ，其中 a 和 m 為常數，且 $0 < m < 1$ 。

(a) 證明 $(1-m^2)y^2 = a^2 - (x-my)^2$ 。(3 分)

(b) 證明當 $y = \frac{x}{m}$ 時 y 值達至最大。(3 分)

(c) 由此決定 x 值 (以 a 和 m 表示) 使 y 值達至最大。(2 分)

4. 在等差數列 $\{a_n\}_{n \geq 1}$ 中，已知 $a_2=3$ 及 $a_{20}=39$ 。

(a) 求數列 $\{a_n\}_{n \geq 1}$ 的通項。(3 分)

(b) 設數列 $\left\{ \frac{1}{a_n a_{n+1}} \right\}_{n \geq 1}$ 的前 n 項和為 S_n 。若 $S_n = \frac{10}{21}$ ，求 n 的值。(5 分)

5. 在 $\triangle ABC$ 中， $\sin(C-A)=1$ 及 $\cos B = \frac{2\sqrt{2}}{3}$ 。

(a) 求 $\sin^2 C$ 。(4 分)

(b) 若 $|AC|=5$ ，求 $\triangle ABC$ 的面積。(4 分)

JM01 數學正卷－參考答案

第一部份 選擇題。

題目編號	最佳答案
1	B
2	E
3	A
4	B
5	B
6	A
7	C
8	D
9	C
10	A
11	B
12	E
13	D
14	A
15	C

(第二部份答案由下頁開始)

第二部份 解答題。

1. (a) 這裏要從 9 本書取出 3 本，因此樣本空間的大小為 ${}_9C_3$ 。事件的大小為 $4 \cdot 2 \cdot 3$ ，故此所求概率為 $\frac{4 \cdot 2 \cdot 3}{{}_9C_3} = \frac{24}{84} = \frac{2}{7}$ 。
- (b) 這裏要排列 9 本書，因此樣本空間的大小為 $9!$ 。此處的事件可視作為將三類書來排列，而中英數三類書各自有 $4!$ 、 $2!$ 、 $3!$ 種方式來排列，故此所求概率為 $\frac{3!4!2!3!}{9!} = \frac{1}{210}$ 。

2. (a) 設 r 為圓的半徑。

方法一

圓的方程是 $(x-4)^2 + (y-4)^2 = r^2$ ----- (1)

把 $y=2x$ 代入 (1)，我們有 $(x-4)^2 + (2x-4)^2 = r^2$ ，即

$$5x^2 - 24x + (32 - r^2) = 0 \text{ ----- (2)}$$

在 L_1 與圓的切點 (即 P)，(2) 的判別式等於零，即

$$(-24)^2 - 4(5)(32 - r^2) = 0 \Leftrightarrow r^2 = \frac{16}{5}。$$

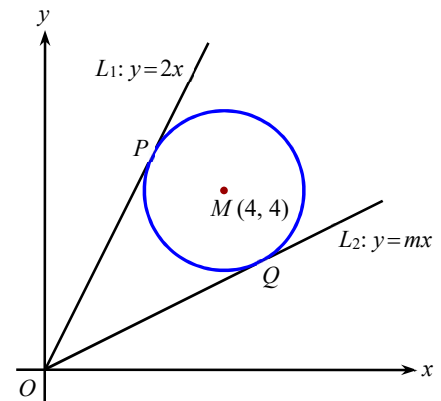
把最後一式代入 (1)，化簡後得圓的方程為

$$5x^2 + 5y^2 - 40x - 40y + 144 = 0 \text{ ----- (3)}$$

方法二

L_1 的方程可寫為 $2x - y = 0$ 。 $\therefore r = |MP| = \frac{|2(4) - 4|}{\sqrt{4+1}} = \frac{4}{\sqrt{5}}$ ，從而圓的方程為

$$(x-4)^2 + (y-4)^2 = \frac{16}{5} \Leftrightarrow 5x^2 + 5y^2 - 40x - 40y + 144 = 0。$$



- (b) 方法一

把 $y=mx$ 代入 (3)，我們有 $5x^2 + 5(mx)^2 - 40x - 40(mx) + 144 = 0$ ，即

$$5(1+m^2)x^2 - 40(1+m)x + 144 = 0 \text{ ----- (4)}$$

在 L_2 與圓的切點 (即 Q)，(4) 的判別式等於零，即 $[-40(1+m)]^2 - 4(5)(1+m^2)(144) = 0$ ，亦即

$$2m^2 - 5m + 2 = 0 \Leftrightarrow (m-2)(2m-1) = 0。對應於直線 L_2 ， $m = \frac{1}{2}$ 。$$

方法二

直線 $y=x$ 穿過 O 和 M 。利用 OP 和 OQ 關於 $y=x$ 的對稱性，即得 $m = \frac{1}{L_1 \text{ 的斜率}} = \frac{1}{2}$ 。

3. (a) 右邊 $= a^2 - (x^2 - 2mxy + m^2y^2) = (a^2 - x^2 + 2mxy) - m^2y^2 = y^2 - m^2y^2 = \text{左邊}$ 。

(b) 由 (a) 的等式得

$$y^2 = \frac{a^2}{1-m^2} - \frac{(x-my)^2}{1-m^2} \text{-----} (1)$$

由於 $0 < m < 1$ ，因此 $0 < m^2 < 1$ ，從而 $\frac{a^2}{1-m^2} \geq 0$ 及 $\frac{(x-my)^2}{1-m^2} \geq 0$ 對任何實數 x, y 都成立。

故此從 (1) 知道對任何實數 x, y 都有 $y^2 \leq \frac{a^2}{1-m^2}$ ，即 $\frac{a^2}{1-m^2}$ 為 y^2 的最大值，並且在 $x-my=0$

($\Leftrightarrow y = \frac{x}{m}$) 時 y^2 達至這個最大值。

$\because y > 0$ ， $\therefore$ 當 $y = \frac{x}{m}$ 時 y 值也達至最大。

(c) 設 $y_{\max}$ 代表 y 的最大值，並設當 $x=x_0$ 時 y 達至最大值。

由 (b) 得知 $y_{\max} = \sqrt{\frac{a^2}{1-m^2}} = \frac{|a|}{\sqrt{1-m^2}}$ ，並且 $x_0 = my_{\max} = \frac{m|a|}{\sqrt{1-m^2}}$ 。

4. (a) 公差 $d = (a_{20} - a_2) / (20 - 2) = 2$ 。

$$\therefore a_1 = a_2 - d = 1。$$

$$\therefore a_n = a_1 + (n-1)d = 2n-1。$$

$$(b) \because S_n = \sum_{k=1}^n \frac{1}{(2k-1)(2k+1)} = \sum_{k=1}^n \frac{1}{2} \left(\frac{1}{2k-1} - \frac{1}{2k+1} \right) = \frac{1}{2} \left(\sum_{k=1}^n \frac{1}{2k-1} - \sum_{k=1}^n \frac{1}{2k+1} \right) = \frac{1}{2} \left(1 - \frac{1}{2n+1} \right) = \frac{n}{2n+1}，$$

$$\therefore S_n = \frac{10}{21} \Rightarrow \frac{n}{2n+1} = \frac{10}{21} \Rightarrow 21n = 20n + 10 \Rightarrow n = 10。$$

5. (a) 由 $\sin(C-A)=1$ 及 $\cos B = \frac{2\sqrt{2}}{3}$ ，得 $A=C-90^\circ$ 及 $\sin B = \frac{1}{3}$ 。

$$\because A+B+C=180^\circ \text{ 及 } A=C-90^\circ, \therefore 2C=270^\circ-B \Rightarrow \cos 2C = \cos(270^\circ-B) = -\sin B = -\frac{1}{3}。$$

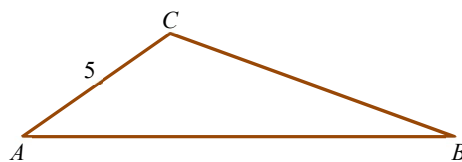
$$\therefore \sin^2 C = \frac{1-\cos 2C}{2} = \frac{1+\frac{1}{3}}{2} = \frac{2}{3}。$$

(b) 由 $C > 90^\circ$ 及 (a) 得 $\cos C = -\sqrt{1-\sin^2 C} = -\frac{\sqrt{3}}{3}$ ，從而有

$$\sin A = \sin(C-90^\circ) = -\cos C = \frac{\sqrt{3}}{3}。$$

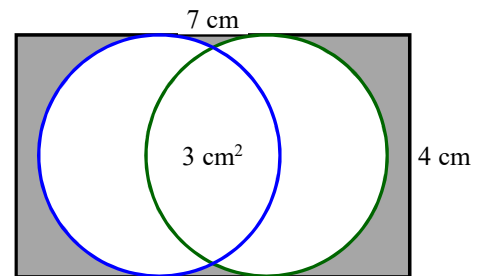
$$\text{由正弦定理得 } BC = \frac{\sin A}{\sin B} AC = \frac{\frac{\sqrt{3}}{3}}{\frac{1}{3}} \cdot 3 \cdot 5 = 5\sqrt{3}。$$

$$\therefore \triangle ABC \text{ 的面積} = \frac{1}{2} AC \cdot BC \cdot \sin C = \frac{1}{2} \cdot 5 \cdot 5\sqrt{3} \cdot \frac{\sqrt{2}}{\sqrt{3}} = \frac{25\sqrt{2}}{2}。$$



Part I Multiple choice questions. Choose the *best answer* for each question.

1. If sets $P = \{1, 2, 3, 5, 7, 11\}$ and $Q = \{x: x^2 - 15x + 36 < 0\}$, then the number of elements in $P \cap Q$ is
A. 2 B. 3 C. 4 D. 5 E. 1
2. Mary and John need respectively 4 hours and 3 hours to finish a certain task. If they work together, how many hours do they need to finish 5 such tasks?
A. $32/5$ B. $35/4$ C. $35/2$ D. $20/3$ E. $60/7$
3. The sum of the first n terms of an arithmetic sequence is n^2 . Find the 10^{th} term of the sequence.
A. 19 B. 21 C. 28 D. 31 E. 40
4. If $y = mx^2 + 6x + 3m$ is positive for any real number x , find the range of m .
A. $0 < m < \sqrt{3}$ B. $m > \sqrt{3}$ C. $-\sqrt{3} < m < \sqrt{3}$
D. $-\sqrt{3} < m < 0$ E. none of the above
5. Suppose α and β are the roots of $-2x^2 + 3x - 7 = 0$. Which of the following equations has $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ as its roots?
A. $x^2 - 3x + 7 = 0$ B. $7x^2 - 3x + 2 = 0$ C. $7x^2 + 3x + 2 = 0$
D. $2x^2 - 3x - 7 = 0$ E. none of the above
6. In the right figure, two identical circles touch the upper and lower edges of the rectangle. The length and width of the rectangle are 7 cm and 4 cm respectively, and the common portion of the two circles has area 3 cm^2 . What is the area (in cm^2) of the shaded region?
A. $31 - 8\pi$ B. $27 - 8\pi$
C. $27 - 4\pi$ D. $21 - 4\pi$
E. none of the above
7. Suppose the equation $9^{-x^2} - 4 \cdot 3^{-x^2} = k$ has a real root. Which of the following must be true?
A. $k > 0$ B. $-4 \leq k \leq 1$ C. $-3 \leq k < 0$ D. $0 < k \leq 3$ E. none of the above
8. Let $f(x) = -16x^3 - mx - m$. If $f(x)$ is divisible by $2x + 1$, find the value of m .
A. -1 B. 1 C. 2 D. 4 E. 6
9. The tens digit (second last digit from the right; e.g. the tens digit of 43128 is 2) of 103^{10} is
A. 2 B. 3 C. 4 D. 7 E. none of the above

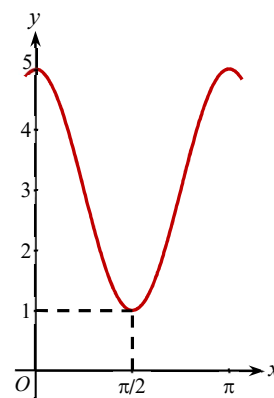


10. If $\log_4 x = y - 3$ and $2(\log_4 x)^2 = 4 - y$, then $x =$

- A. $\frac{1}{4}$ or 2 B. $\frac{1}{2}$ or 4 C. $\frac{7}{2}$ or 2 D. $\frac{1}{4}$ or $\frac{7}{2}$ E. 2 or 4

11. The right figure shows the graph of _____.

- A. $y = 3 + 2 \cos \frac{x}{2}$
 B. $y = 3 + 2 \cos 2x$
 C. $y = 3 + 2 \cos x$
 D. $y = 1 + 2 \cos \frac{x}{2}$
 E. $y = 1 + 2 \cos 2x$



12. $P(-1, -3)$ and $Q(5, -1)$ are two given points. The equation of the perpendicular bisector of PQ is

- A. $x + 3y - 4 = 0$ B. $x - 3y + 4 = 0$ C. $x + 3y + 4 = 0$
 D. $3x - y - 4 = 0$ E. $3x + y - 4 = 0$

13. For the data set $x_1, x_2, \dots, x_n$, the mean and the variance are respectively 1 and 0.01. The mean and the variance for the data set $10x_1, 10x_2, \dots, 10x_n$ are respectively

- A. 1 and 0.01 B. 10 and 0.1 C. 1 and 1 D. 10 and 1 E. 100 and 1

14. $\frac{\sqrt{140} - \sqrt{132}}{\sqrt{35} + \sqrt{33}} =$

- A. $68 - 2\sqrt{1155}$ B. $68 - \sqrt{1155}$ C. $(34 - \sqrt{1155})/2$
 D. $34 - \sqrt{1155}$ E. $68 + 2\sqrt{1155}$

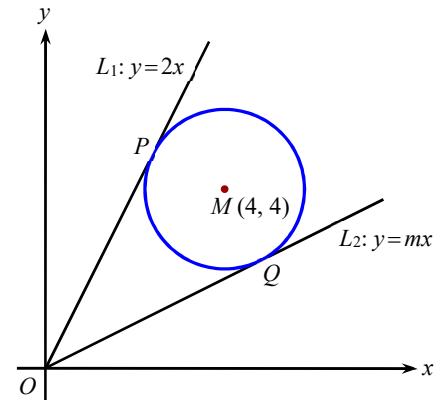
15. If $\frac{5}{a} + \frac{4}{b} = 3$ ($a, b > 0$), then the minimum value of ab is

- A. $\frac{20}{9}$ B. $\frac{20}{3}$ C. $\frac{80}{9}$ D. $\frac{80}{3}$ E. $\frac{\sqrt{20}}{3}$

Part II Problem-solving questions.

1. There are 4 Chinese books, 2 English books, and 3 Mathematics books on a bookshelf.
- (a) Three books are randomly chosen from the bookshelf. Find the probability that one Chinese book, one English book, and one Mathematics book are chosen. (3 marks)
- (b) These nine books are randomly re-arranged. Find the probability that books of the same kind are put together. (5 marks)
- [Note: Write the answers of (a) and (b) as fractions in the lowest terms.]

2. In the right figure, the two lines $L_1: y = 2x$ and $L_2: y = mx$ are tangent to a circle centered at $M(4, 4)$ at P and Q respectively.
- (a) Find the equation of the circle. (4 marks)
- (b) Find the value of m . (4 marks)



3. Suppose $x, y > 0$ and $y^2 - 2myx + x^2 = a^2$, where a and m are constants with $0 < m < 1$.
- (a) Show that $(1 - m^2)y^2 = a^2 - (x - my)^2$. (3 marks)
- (b) Show that y attains its maximum value when $y = \frac{x}{m}$. (3 marks)
- (c) Hence determine the value of x (in terms of a and m) that maximizes y . (2 marks)
4. For the arithmetic sequence $\{a_n\}_{n \geq 1}$, $a_2 = 3$ and $a_{20} = 39$.
- (a) Find the general term of $\{a_n\}_{n \geq 1}$. (3 marks)
- (b) Let S_n be the sum of the first n terms of sequence $\left\{ \frac{1}{a_n a_{n+1}} \right\}_{n \geq 1}$. If $S_n = \frac{10}{21}$, find the value of n . (5 marks)
5. In $\triangle ABC$, $\sin(C - A) = 1$ and $\cos B = \frac{2\sqrt{2}}{3}$.
- (a) Find $\sin^2 C$. (4 marks)
- (b) If $|AC| = 5$, find the area of $\triangle ABC$. (4 marks)

JM01 Mathematics Standard – Suggested Answer

Part I Multiple choice questions.

Question Number	Best Answer
1	B
2	E
3	A
4	B
5	B
6	A
7	C
8	D
9	C
10	A
11	B
12	E
13	D
14	A
15	C

(Answers for Part II start from next page)

Part II Problem-solving questions.

1. (a) Here three books are chosen from nine, and so the size of the sample space is ${}_9C_3$. The size of the event is $4 \cdot 2 \cdot 3$, and hence the required probability is $\frac{4 \cdot 2 \cdot 3}{{}_9C_3} = \frac{24}{84} = \frac{2}{7}$.
- (b) Here we want to permute 9 books, and so the size of the sample space is $9!$. The event can be regarded as a permutation of three groups, and there are $4!$ ways, $2!$ ways, and $3!$ ways to permute the Chinese books, English books, and Mathematics books respectively. It follows that the required probability is $\frac{3!4!2!3!}{9!} = \frac{1}{210}$.

2. (a) Let r be the radius of the circle.

Method 1

Equation of the circle is $(x-4)^2 + (y-4)^2 = r^2$ ----- (1)

Putting $y=2x$ into (1), we have $(x-4)^2 + (2x-4)^2 = r^2$,
i.e.

$$5x^2 - 24x + (32 - r^2) = 0 \text{ ----- (2)}$$

At the tangent point P , the discriminant of (2) is zero, i.e.

$$(-24)^2 - 4(5)(32 - r^2) = 0 \Leftrightarrow r^2 = \frac{16}{5}.$$

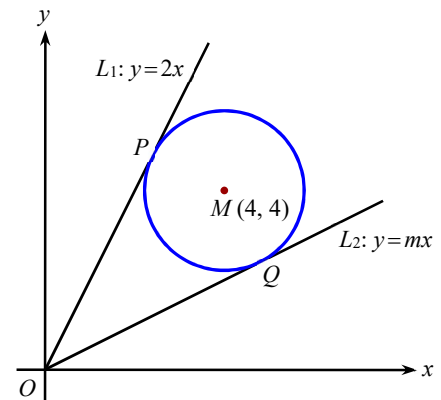
Putting the last equation into (1), upon simplification, we get the equation of the circle:

$$5x^2 + 5y^2 - 40x - 40y + 144 = 0 \text{ ----- (3)}$$

Method 2

Equation of L_1 can be written as $2x - y = 0$. $\therefore r = |MP| = \frac{|2(4) - 4|}{\sqrt{4+1}} = \frac{4}{\sqrt{5}}$, and so the equation of the circle is

$$(x-4)^2 + (y-4)^2 = \frac{16}{5} \Leftrightarrow 5x^2 + 5y^2 - 40x - 40y + 144 = 0.$$



- (b) Method 1

Putting $y=mx$ into (3), we have $5x^2 + 5(mx)^2 - 40x - 40(mx) + 144 = 0$, i.e.

$$5(1+m^2)x^2 - 40(1+m)x + 144 = 0 \text{ ----- (4)}$$

At the tangent point Q , the discriminant of (4) is zero, i.e. $[-40(1+m)]^2 - 4(5)(1+m^2)(144) = 0 \Leftrightarrow 2m^2 - 5m + 2 = 0 \Leftrightarrow (m-2)(2m-1) = 0$. Corresponding to L_2 , $m = \frac{1}{2}$.

Method 2

The line $y=x$ passes through O and M . Symmetry of OP and OQ about $y=x$ yields $m = \frac{1}{\text{Slope of } L_1} = \frac{1}{2}$.

3. (a) $\text{RHS} = a^2 - (x^2 - 2mxy + m^2y^2) = (a^2 - x^2 + 2mxy) - m^2y^2 = y^2 - m^2y^2 = \text{LHS}.$

(b) From the equality of (a) we have

$$y^2 = \frac{a^2}{1-m^2} - \frac{(x-my)^2}{1-m^2} \text{ ----- (1)}$$

Since $0 < m < 1$, we have $0 < m^2 < 1$, and so $\frac{a^2}{1-m^2} \geq 0$ and $\frac{(x-my)^2}{1-m^2} \geq 0$ is true for all $x, y \in \mathbb{R}$.

It follows from (1) that for all $x, y \in \mathbb{R}$, $y^2 \leq \frac{a^2}{1-m^2}$, i.e. $\frac{a^2}{1-m^2}$ is the maximum value of y^2 , and y^2 attains this maximum value when $x-my=0$ ($\Leftrightarrow y = \frac{x}{m}$).

$\therefore y > 0$, $\therefore y$ also attains this maximum value when $y = \frac{x}{m}$.

(c) Let $y_{\max}$ represent the maximum value of y , and let y attain this maximum value when $x=x_0$.

From (b), $y_{\max} = \sqrt{\frac{a^2}{1-m^2}} = \frac{|a|}{\sqrt{1-m^2}}$ and $x_0 = my_{\max} = \frac{m|a|}{\sqrt{1-m^2}}.$

4. (a) Common difference $d = (a_{20} - a_2)/(20 - 2) = 2.$

$\therefore a_1 = a_2 - d = 1.$

$\therefore a_n = a_1 + (n-1)d = 2n - 1.$

(b) $\therefore S_n = \sum_{k=1}^n \frac{1}{(2k-1)(2k+1)} = \sum_{k=1}^n \frac{1}{2} \left(\frac{1}{2k-1} - \frac{1}{2k+1} \right) = \frac{1}{2} \left(\sum_{k=1}^n \frac{1}{2k-1} - \sum_{k=1}^n \frac{1}{2k+1} \right) = \frac{1}{2} \left(1 - \frac{1}{2n+1} \right) = \frac{n}{2n+1},$

$\therefore S_n = \frac{10}{21} \Rightarrow \frac{n}{2n+1} = \frac{10}{21} \Rightarrow 21n = 20n + 10 \Rightarrow n = 10.$

5. (a) From $\sin(C - A) = 1$ and $\cos B = \frac{2\sqrt{2}}{3}$, we have $A = C - 90^\circ$ and $\sin B = \frac{1}{3}.$

$\therefore A + B + C = 180^\circ$ and $A = C - 90^\circ$, $\therefore 2C = 270^\circ - B \Rightarrow \cos 2C = \cos(270^\circ - B) = -\sin B = -\frac{1}{3}.$

$\therefore \sin^2 C = \frac{1 - \cos 2C}{2} = \frac{1}{2} \left(1 + \frac{1}{3} \right) = \frac{2}{3}.$

(b) From $C > 90^\circ$ and (a), we get $\cos C = -\sqrt{1 - \sin^2 C} = -\frac{\sqrt{3}}{3},$

and so $\sin A = \sin(C - 90^\circ) = -\cos C = \frac{\sqrt{3}}{3}.$

Law of Sines yields $BC = \frac{\sin A}{\sin B} AC = \frac{\sqrt{3}}{3} \cdot 3 \cdot 5 = 5\sqrt{3}.$

$\therefore \text{area of } \triangle ABC = \frac{1}{2} AC \cdot BC \cdot \sin C = \frac{1}{2} \cdot 5 \cdot 5\sqrt{3} \cdot \frac{\sqrt{2}}{\sqrt{3}} = \frac{25\sqrt{2}}{2}.$

