



A Novel Text Encryption Algorithm based on Core Adaptive Fourier Decomposition

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ABSTRACT

The progress of modern science and technology has accelerated the development of communication, providing more focus on the security of data transmission. Although there are many recognized encryption techniques, more high-performance algorithms are required to withstand the sophisticated analysis and cracking techniques. In this paper, we propose an innovative algorithm that combines with Core Adaptive Fourier Decomposition (Core-AFD) for text encryption. In addition, two versions of algorithm, initial version and enhanced version, are discussed for different forms and strength of the secret key and ciphertext. Due to the one-to-one correspondence between the decomposition and the error terms, the encryption algorithm is theoretically feasible and safe. The algorithm is further demonstrated by conducting experiments on text encryption with different lengths. The results are promising. Finally, the time complexity of cracking our algorithm in different cases is given.

CCS CONCEPTS

• Security and privacy → Software and application security
→ Domain-specific security and privacy architectures • Security and privacy → Cryptography → Information-theoretic techniques

KEYWORDS

Cryptography, Text Encryption, Adaptive Fourier Decomposition, Signal Processing

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1 Introduction

Cryptography converts important plain text into unreadable ciphertext to prevent intruders from intercepting them. It used to involve the diplomatic and military fields, but due to the rapid development of network technology, it has recently been applied to many areas of real life. Various encryption approaches using different techniques have been proposed over the years. Some classic and effective encryption algorithms (Data Encryption Standard, DES; Advanced Encryption Standard, AES and Rivest-Shamir-Adleman, RSA [1]-[3]) are widely used in the cryptography area. Since 1990s, some researchers have found that maybe one can establish relationships between cryptography and chaos because chaos have the characteristics of non-periodicity, randomness and sensitivity to initial conditions [4]. A plenty of encryption methods using chaos have been developed.

In the field of signal analysis, the positive frequency representation of signal is always a hot topic. Based on the theory of mono-component function, a series of Adaptive Fourier Decomposition (AFD) methods have been invented, including Core-AFD, Unwinding AFD and Cyclic AFD [5]-[7]. They have been applied in many aspects like ECG signal classification, image reconstruction and system identification because of its fast convergence which has been proved.

In this paper, we discuss how to combine encryption with signal decomposition method and propose a novel text encryption algorithm based on Core-AFD. Relations of ciphertexts to both the plaintext and secret key should be weakened in an excellent encryption method. As the feature of one-to-one correspondence between decomposition and error term, it would be feasible and

effective to set them as the cyphertext and secret key respectively. Taking encryption strength into consideration, two versions of our encryption algorithms are displayed. The difference is that there is a variable parameter in the enhanced version, making it more secure and harder to be attacked.

The rest of this paper is organized as follows. Section 2 gives a brief introduction of Core-AFD. Our proposed encryption algorithm in two versions and the decryption process are presented in Section 3. Section 4 displays the experimental results of our algorithm. In Section 5, the security of our method is analyzed. Section 6 shows conclusions.

2 Preliminaries

Motivated by research interest on positive instantaneous frequency of signals, a series of AFD algorithm has been developed. AFD obtains the positive frequency expansion of analytic rational function in the case of simple and complex variables functions. Additionally, its fast convergence is also proved.

2.1 Takenaka-Malmquist system [8]

The Takenaka-Malmquist system is also called the rational orthogonal system, which is an important component of AFD. Such rational fractions are fundamental modules to rational functions in Hardy spaces H^2 . In complex plane \mathbb{C} , $\{B_k\}_{k=1}^\infty$ the Takenaka-Malmquist system generated by $a_k \in \mathbb{D}$, where \mathbb{D} denotes the unit disc, is defined by

$$B_k(z) = B_{\{a_1, \dots, a_k\}}(z) = \frac{\sqrt{1 - |a_k|^2}}{1 - \bar{a}_k z} \prod_{l=1}^{k-1} \frac{z - a_l}{1 - \bar{a}_l z}. \quad (1)$$

The Takenaka-Malmquist system consists of functions with positive frequencies which can be seen from (1).

It has been demonstrated with long-term studies that the system is complete in disc algebra $A(\mathbb{D})$ and Hardy p spaces $H^p(\mathbb{D})$, $1 \leq p \leq \infty$, if and only if the below Szaász condition satisfies (see [9], [10])

$$\sum_{k=1}^{\infty} (1 - |a_k|) = \infty. \quad (2)$$

2.2 Core-AFD [5]

As opposed to traditional Takenaka-Malmquist system, Core-AFD holds excellent adaptability, providing an approach to decompose complex-value analytic functions in complex H^2 and general real-valued functions in Lebesgue L^2 spaces. This paper concentrates on real-valued functions.

Denote by F the real-valued function with finite energy on the unit circle $\partial\mathbb{D}$. Since it can be divided into F^+ and F^- in the form of direct sum decomposition

$$\begin{aligned} F(e^{it}) &= \sum_{k=-\infty}^{\infty} \rho_k e^{ikt} \\ &= \sum_{k=0}^{\infty} \rho_k e^{ikt} + \sum_{k=-\infty}^{-1} \rho_k e^{ikt} \\ &= F^+(e^{it}) + F^-(e^{it}), \end{aligned} \quad (3)$$

where $\rho_k = \frac{1}{2\pi} \int_0^{2\pi} F(e^{it}) e^{-ikt} dt$ holds the equation that $\rho_{-k} = \bar{\rho}_k$, then there is

$$F(e^{it}) = -\rho_0 + 2\text{Re}F^+(e^{it}). \quad (4)$$

Here F^+ is the projection of F onto H^2 . Therefore, the problem can be reduced to the decomposition of F^+ .

2.2.1 Szegő kernel [11]

Core-AFD introduces unit module of the unit disc, Szegő kernel e_a , whose linear combination is contained in H^2 .

$$e_a(z) = \frac{\sqrt{1 - |a|^2}}{1 - \bar{a}z}, a \in \mathbb{D}. \quad (5)$$

Drawing lessons from back shift operator, we rewrite F^+ as follows

$$\begin{aligned} F^+(z) &= F_1^+(z) = \langle F_1^+, e_{a_1} \rangle e_{a_1}(z) + \frac{F_1^+(z) - \langle F_1^+, e_{a_1} \rangle e_{a_1}(z)}{\frac{z - a_1}{1 - \bar{a}_1 z}} \\ &= \langle F_1^+, e_{a_1} \rangle e_{a_1}(z) + R_1(z), \end{aligned} \quad (6)$$

where a_1 can be any complex number in unit disc. Denote by

$$F_2^+(z) = \frac{F_1^+(z) - \langle F_1^+, e_{a_1} \rangle e_{a_1}(z)}{\frac{z - a_1}{1 - \bar{a}_1 z}}, \text{ then there holds} \quad (7)$$

$$R_1(z) = F_2^+(z) \frac{z - a_1}{1 - \bar{a}_1 z}.$$

So (6) is identical with the below equation (8)

$$F^+(z) = \langle F_1^+, e_{a_1} \rangle e_{a_1}(z) + F_2^+(z) \frac{z - a_1}{1 - \bar{a}_1 z}. \quad (8)$$

Repeating the process until $F_n^+(z)$ by recursive formula

$$F_{k+1}^+(z) = (F_k^+(z) - \langle F_k^+, e_{a_k} \rangle e_{a_k}) \frac{1 - \bar{a}_k z}{z - a_k}, \quad (9)$$

we will obtain the decomposition formula

$$F^+(z) = \sum_{k=1}^n \langle F_k^+, e_{a_k} \rangle B_k(z) + F_{k+1}^+(z) \prod_{k=1}^n \frac{z - a_k}{1 - \bar{a}_k z}, \quad (10)$$

where the remainder term after k -th decomposition is expressed as

$$R_k(z) = F_{k+1}^+(z) \prod_{k=1}^n \frac{z - a_k}{1 - \bar{a}_k z}. \quad (11)$$

2.2.2 Maximal Selection Principle (MSP) [12]

In addition to the desire to construct a positive instantaneous frequency decomposition system with an explicit expression like (10), it is also eager for Core-AFD that the instantaneous frequency be increased in accordance with the decomposition hierarchy. That is the reason it applies MSP to select applicable $a_k, k = 1, \dots, n$.

Szegő kernel is the Cauchy kernel in arc length measure so that it has the nature of reproducing kernel [13]. It is due to the orthogonality of Hilbert space and unit modular property of complex number for Möbius transformation [14], we can derive the below energy relation from (8)

$$|F^+|^2 = |\langle F_1^+, e_{a_1} \rangle e_{a_1}|^2 + |F_2^+|^2 = (1 - |a_1|^2) |F_1^+(a_1)|^2 + |F_2^+|^2. \quad (12)$$

To extract the maximum energy from the first decomposition $\langle F_1^+, e_{a_1} \rangle e_{a_1}(z)$, our purpose can be converted to maximize $(1 - |a_1|^2) |F_1^+(a_1)|^2$ in the range of unit disc \mathbb{D} , we have

$$a_1 = \arg \max \{ (1 - |a|^2) |F_1^+(a)|^2 : a \in \mathbb{D} \}. \quad (13)$$

We perform this maximum selection in a similar way to get all the befitting a_k for (10)

$$a_k = \arg \max \{ (1 - |a|^2) |F_k^+(a)|^2 : a \in \mathbb{D} \}. \quad (14)$$

The decomposition process is a consecutive energy approximation which means in the L^2 -norm sense,

$$F^+(z) = \sum_{k=1}^{\infty} \langle F_k^+, e_{a_k} \rangle B_k(z). \quad (15)$$

For a proof, it can refer to [15] and [16].

3 Proposed Encryption Algorithm

Our proposed text encryption algorithm adopts a novel thought of signal decomposition, portending it quite different from existing encryption methods, nonetheless, its encryption performance not compromised.

The two most crucial parts of cryptography are the secret key and ciphertext. It is critical to choose the secret key and ciphertext appropriate for Core-AFD so that it could not only meet the fundamental characteristics, but also can guarantee a splendid encryption and decryption property. Inspired by the relationship, one-to-one correspondence, between error and decomposition term, we found the feasibility to employ them as the secret key and ciphertext respectively. The flowchart of two versions encryption process is shown in Fig. 1.

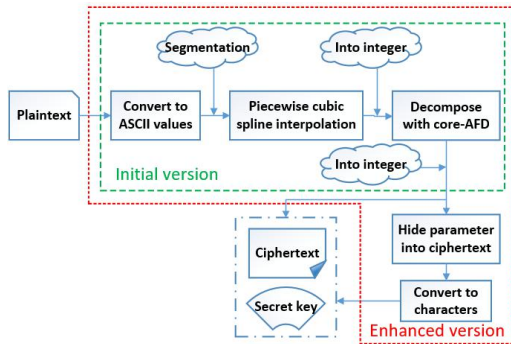


Figure 1: The flowchart of two versions encryption process: 1) initial version in green box; 2) enhanced version in red box.

3.1 Re-processing

The common operation to process characters in text files is to convert them into ASCII values, which implies the targets we will face become a series of integers with distinct numerical difference. There are risks and costs if integers of this type are processed directly with Core-AFD. Since almost every integer forms a peak or valley in the curve, it will need to increase the number of the decomposition for anticipated outcomes. Otherwise, the error will be large enough to affect encryption and decryption. Thus, smoothing is required before applying Core-AFD to converted integers.

There are plenty of smoothing methods available and may have a positive impact on our encryption algorithm. In this paper, we select piecewise cubic spline interpolation as the smoothing method for the listed five reasons:

- (1) Calculation is simple and easy to implement;
- (2) Function is continuous;
- (3) Generate smooth curve;
- (4) Curvature changes continuously;
- (5) Degree of polynomial is low.

3.2 Encryption Algorithm

Denote by T the converted ASCII values of the plaintext, N the length of the whole plaintext. Considering the computation of long text processing, segmentation for the plaintext is introduced. Set η as an appropriate threshold for the length of text. Then we will obtain the subsection texts for encryption, expressing as

$$\tilde{T} = \begin{cases} T_i, N > \eta \\ T, N \leq \eta \end{cases}, \quad (16)$$

where $i = 1, \dots, m$, $m = \lceil N/\eta \rceil$. The length of T_k , $k = 1, \dots, m-1$ is $\lceil N/m \rceil$. Here, symbol ' \lceil ' means rounding up to an integer and symbol ' \lfloor ' expresses to round to the nearest integers towards minus infinity.

As mentioned in Section 3.1, the piecewise cubic spline interpolation is executed for the \tilde{T} to acquire smooth and continuous function, denoting as f after rounding each value to the nearest integer. After that, decompose real integer function f in the form of Core-AFD with proper number of decomposition M . The partial sum based on projection $f^+ \in H^2$ of f can be decomposed as

$$\tilde{f}^+(z) = \sum_{k=1}^M \langle f_k^+, e_{a_k} \rangle B_k(z), \quad (17)$$

then we will obtain

$$\tilde{f} = -\rho_0 + 2\text{Re}\tilde{f}^+, \quad (18)$$

where $\rho_0 = \frac{1}{2\pi} \int_0^{2\pi} f(e^{it}) dt$, and a_k is ensured by MSP.

3.2.1 Initial Version

In order to correspond to ASCII values, ranging from 0 to 126 in decimal numeral system, it is required to round up every value of \tilde{f} to an integer. The rest points of the smoothed function are recorded as the ciphertext $C^{(1)}$ when removing all the interpolation ones. The initial secret key $K^{(1)}$ at this encryption consists of all non-interpolation points of $f - \lceil \tilde{f} \rceil$. In this initial encryption version, both the secret key and ciphertext are presented as integers. They are not converted into characters by ASCII because there are probably some values not between 0 and 126. Our proposed text encryption algorithms of initial version based on Core-AFD is as Algorithm 1.

Algorithm 1: Initial version

Input: plaintext, length of the plaintext N , segmentation threshold η and number of the decomposition M .

Output: secret key $K^{(1)}$ and ciphertext $C^{(1)}$.

- 1: Convert the plaintext into corresponding ASCII values T .
- 2: **if** $N > \eta$ **do**
- 3: Segment the plaintext, $\tilde{T} = T_i, i = 1, \dots, \lfloor N/\eta \rfloor$;
- 4: **else** No segmentation, $\tilde{T} = T$;
- 5: **end if**
- 6: Execute piecewise cubic spline interpolation for \tilde{T} ;
- 7: Round values of smoothed function to nearest integer, denoted by f ;
- 8: Decompose f with Core-AFD:

$$\text{Get } \tilde{f}^+(z) = \sum_{k=1}^M \langle f_k^+, e_{a_k} \rangle B_k(z);$$

$$\text{Get } \tilde{f} = -\rho_0 + 2\text{Re}\tilde{f}^+;$$

- 9: **return** $C^{(1)}$ from $\lceil \tilde{f} \rceil$ and $K^{(1)}$ from $f - \lceil \tilde{f} \rceil$.
-

3.2.2 Enhanced Version

However, further process to initial secret key is considered for visualization, at the same time, improving encryption performance and difficulty of decryption. We introduce β which ensures all values of $K^{(2)}$ are ASCII values within 0 and 126, to obtain every $K_i^{(2)}$ of the new secret key $K^{(2)}$.

$$K_i^{(2)} = K_i^{(1)} + \beta - \min_{1 \leq k \leq N} K_k^{(1)}. \quad (19)$$

As $\beta - \min_{1 \leq k \leq N} K_k^{(1)}$ is necessary in decryption process, it is hidden in the ciphertext, forming the new ciphertext $C^{(2)}$.

$$C^{(2)} = \{ \beta - \min_{1 \leq k \leq N} K_k^{(1)}, C^{(1)} \}. \quad (20)$$

Then we can make the secret key and ciphertext appear as characters as possible by transforming them with ASCII. To simplify, still denote them by $K^{(2)}$ and $C^{(2)}$. The enhanced version of our algorithm is as Algorithm 2.

Algorithm 2: Enhanced version

Input: plaintext, β , length of the plaintext N , segmentation threshold η and number of the decomposition M .

Output: secret key $K^{(2)}$ and ciphertext $C^{(2)}$.

- 1: Convert the plaintext into corresponding ASCII values T .
 - 2: **if** $N > \eta$ **do**
 - 3: Segment the plaintext, $\tilde{T} = T_i, i = 1, \dots, \lfloor N/\eta \rfloor$;
 - 4: **else** No segmentation, $\tilde{T} = T$;
 - 5: **end if**
 - 6: Execute piecewise cubic spline interpolation for \tilde{T} ;
 - 7: Round values of smoothed function to nearest integer, denoted by f ;
 - 8: Decompose f with Core-AFD:
 - Get $\tilde{f}^+(z) = \sum_{k=1}^M \langle f_k^+, e_{a_k} \rangle B_k(z)$;
 - Get $\tilde{f} = -\rho_0 + 2\text{Re}\tilde{f}^+$;
 - 9: Get $C^{(1)}$ from $\lceil \tilde{f} \rceil$ and $K^{(1)}$ from $f - \lceil \tilde{f} \rceil$;
 - 10: Get $K_i^{(2)} = K_i^{(1)} + \beta - \min_{1 \leq k \leq N} K_k^{(1)}$;
 - Get $C^{(2)} = \{ \beta - \min_{1 \leq k \leq N} K_k^{(1)}, C^{(1)} \}$;
 - 11: **return** $C^{(2)}$ and $K^{(2)} = \{ K_i^{(2)}, i = 1, \dots, N \}$.
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3.3 Decryption Process

The principle of decryption is simple when grasping the encryption process well. We will display the decryption methods of two encryption versions respectively.

- Decryption of initial version

In the initial encryption algorithm, the secret key and ciphertext are presented as integers rather than characters. They correspond to the decomposition term and the error term, whose lengths are the same as that of the plaintext. It is just a simple sum that will restore the original signal. Then the decrypted plaintext \hat{T} is obtained as

$$\hat{T} = C^{(1)} + K^{(1)}. \quad (21)$$

- Decryption of enhanced version

Although it will be slightly complex when proceeding the decryption for enhanced encryption algorithm, the security of enhanced version is higher. By default, both sides of communication know where $\beta - \min_{1 \leq k \leq N} K_k^{(1)}$ is placed in the ciphertext so that it can be extracted to make sure the length of the ciphertext consistent with the plaintext and secret key. It is noticed that $C^{(1)}$ is the value after removing $\beta - \min_{1 \leq k \leq N} K_k^{(1)}$ from

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